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HETEROCLINIC ORBITS AND ROTATION SETS FOR TWIST MAPS

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Abstract. We show how the shadowing property can be used in connection to rotation sets. We review the concept of periodic chains and the shadowing rotation property, and study a class of diffeomorphisms with invariant sets that have such property. In particular, we consider invariant sets that arise from homoclinic and heteroclinic connections for twist maps in higher dimensions. As a consequence, we can show the existence of a family of twist maps, each one with an open set of rotation vectors that are realized by points that are close to a fixed point.

1. **Introduction.** The concept of rotation numbers, introduced by Poincaré, has been used to understand the dynamics of orientation preserving homeomorphisms of the circle and has allowed a useful characterization of them. We have two different behaviors depending on whether the corresponding rotation number is rational or irrational.

A similar idea has been used for annulus homeomorphisms. In this case, we get a set of different rotation numbers called the rotation set. Similar to the circle, there is a correspondence between the different rotation numbers and the dynamics of the homeomorphisms. This correspondence is even more transparent in the case of *twist* maps. This has become a active research area in recent years. The cornerstones of the theory are the Aubry-Mather and the Poincaré-Birkhoff theorems. See for example: [7, 10, 13, 15]. Other generalizations of rotation sets have been proposed. For example, for torus homeomorphisms, the paper of Misiurewicz and Ziemian [24] gives a summary of three different generalizations and their relations.

One of the main difficulties that arises when working with the different generalizations, is the question of the meaning of the sets obtained. For monotone twist maps in one dimension each rational rotation number represents a periodic orbit that actually is *realized* by a point, i.e.: there exists at least one periodic point with that rotation number.

The non-twist situation was investigated by Aronson et al [2] and they found that under certain conditions the existence of an homoclinic point for a fixed point of a diffeomorphism of the annulus implies the existence of an open set of rotation

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