Basic facts on topological bifurcation theory

Slawomir Rybicki
Nicolaus Copernicus University
Torun, Poland

Many of the problems of mathematics and mechanics reduces to the study of zeros of continuous families of mappings $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$. Bifurcation theory is a branch of mathematics investigating a structure of the set of solutions of the equation

$$f(x, \lambda) = 0 \tag{E}$$

under changing parameter $\lambda \in \mathbb{R}$.

More precisely speaking, assume that $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuous map such that $f(0, \lambda) = 0$ for all $\lambda \in \mathbb{R}$. The purpose of my talk is to study solutions of (E) satisfying $x \neq 0$.

A point $(0, \lambda_0) \in \mathbb{R}^n \times \mathbb{R}$ is said to be a bifurcation point of solutions of (E) if any sufficiently small neighborhood of $(0, \lambda_0)$ contains a solution (x, λ_1) such that $x \neq 0$.

We are going to formulate necessary and sufficient conditions for the existence of bifurcation points of solutions of (E).

Finally we will formulate some versions of the famous Krasnosel'skii local bifurcation theorem and Rabinowitz' global bifurcation theorem.