CÁLCULO DIFERENCIAL E INTEGRAL II
Laboratorio 16
Otoño 2021
Sucesiones. Series numéricas y criterios de convergencia. Convergencia absoluta y condicional

1. Calcula el límite de cada sucesión $\{a_n\}$ o justifica si ésta diverge:

   (a) $a_n = \dfrac{(n+1)(n+2)}{2n^2}$.

   (b) $a_n = \sqrt{n+\sqrt{n}} - \sqrt{n-\sqrt{n}}$.

   (c) $a_n = \left(\dfrac{n+1}{2n}\right)\left(1-\dfrac{1}{n}\right)$.

   (d) $a_n = \left(\dfrac{n+3}{n+1}\right)^n$.

   (e) $a_n = \dfrac{(-1)^n+1}{n}$.

   (f) $a_n = (-1)^n \dfrac{n}{n+1}$.

   (g) $a_n = (\ln n)^{1/n}$. Sugerencia: $1 \le \ln n \le n$, para $n \ge 3$.

2. Demuestra que:

   (a) $\lim\limits_{n\to\infty} (1+\alpha^n)^{1/n} = \begin{cases} 1, & \text{si } 0<\alpha\le 1, \\ \alpha & \text{si } \alpha > 1. \end{cases}$

   (b) $\lim\limits_{n\to\infty} \sqrt[n]{a^n+b^n} = \max(a,b),\ \ a,b>0.$

3. Calcula las siguientes series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n}{3^{2n-1}}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+2}+3^{n+1}}{6^n}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n+1]{n+1} - \sqrt[n]{n}\right)$.

   (d) $\displaystyle\sum_{n=0}^{\infty} \int_{n+1}^{n+2} \frac{1}{1+x^2}\,dx$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. Sugerencia: usa fracciones parciales.

4. Estudia la convergencia de las siguientes series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{7}\right)^{1/n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}\left(1-\frac{1}{n}\right)^{n^2}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{3!\ n!\ 3^n}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n+n}{n!}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n+1}$.

   (f) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$.

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$.

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}+\sqrt[3]{n}}$.

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$.

   (j) $\displaystyle\sum_{n=1}^{\infty} \tan\left(\frac{1}{2^n}\right)$.

5. Analiza si las siguientes series convergen absolutamente, convergen condicionalmente o divergen:

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{\sqrt{n^3}}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{n}{n^2+1}$.

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{\operatorname{sen}(n)}{n^2}$.

   (d) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\left(1-\frac{2}{n}\right)^{n^2}$.