CÁLCULO DIFERENCIAL E INTEGRAL II
Laboratorio 13
Otoño 2021
Integrales impropias

1. Calcula la integral impropia o muestra que diverge:

   (a) $\displaystyle\int_1^{\infty} \frac{dx}{x(1+5x)}.$

   (b) $\displaystyle\int_{\ln 2}^{\infty} \frac{e^{-x}}{1-e^{-2x}} dx.$

   (c) $\displaystyle\int_0^{\infty} \frac{1}{e^x+e^{-x}} dx.$

   (d) $\displaystyle\int_{-\infty}^{0} xe^{2x}\ dx.$

   (e) $\displaystyle\int_{-\infty}^{\infty} |x|\, e^{-x^2} dx.$

   (f) $\displaystyle\int_0^1 x\ln(x) dx.$

   (g) $\displaystyle\int_0^{\pi/2} \frac{dx}{1-\operatorname{sen}(x)}.$

   (h) $\displaystyle\int_0^1 \frac{e^x}{e^x-1} dx.$

   (i) $\displaystyle\int_0^1 \frac{4r}{\sqrt{1-r^4}} dr.$

   (j) $\displaystyle\int_1^{\cosh(t)} \frac{dx}{\sqrt{x^2-1}},\ t \geq 0.$

   (k) $\displaystyle\int_0^2 \frac{dx}{\sqrt{|x-1|}}.$

   (l) $\displaystyle\int_a^b \frac{dx}{\sqrt{x-a}\sqrt{b-x}}, \quad a < b$ dados.

2. Demuestra que
$$\int_0^1 (\ln x)^n\, dx = (-1)^n n!$$

3. Deduce cuál debe ser el valor de la constante A para el cual converge la siguente integral impropia. ¿A qué converge la integral?:
$$\int_2^{\infty} \left(\frac{x}{2x^2+1} - \frac{A}{x+1} \right) dx.$$

4. Utiliza algún criterio de convergencia para determinar si la integral impropia converge o diverge:

   (a) $\displaystyle\int_1^\infty \frac{2x^2+1}{x^4+2x+1}\,dx.$

   (b) $\displaystyle\int_2^\infty \frac{dx}{(1+x)\ln x}.$

   (c) $\displaystyle\int_1^\infty \frac{1}{1+x^{1/2}}dx.$

   (d) $\displaystyle\int_2^\infty \frac{\sqrt{x^2+1}}{x}dx.$

   (e) $\displaystyle\int_1^\infty \frac{x}{e^{2x}-1}dx.$

   (f) $\displaystyle\int_0^\infty \frac{\tan^{-1} x}{1+x^4}\,dx.$

   (g) $\displaystyle\int_0^\infty e^{-x^2}\,dx.$

   (h) $\displaystyle\int_0^1 \frac{\operatorname{sen}(x)}{\sqrt{x}}\,dx.$

   (i) $\displaystyle\int_0^1 e^{1/x}\,dx.$

   (j) $\displaystyle\int_0^\infty \frac{|\operatorname{sen}(x)|}{x^{3/2}}\,dx.$

   (k) $\displaystyle\int_3^\infty \frac{\ln x}{(x-3)^4}\,dx.$