SEMINARIO DE MATEMÁTICAS

On Dirichlet and Schwarz problems for generalized Beltrami equations

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Abstract

The linear sytem of first-order partial differential equations for real-valued functions u and v

$$\begin{aligned} a_{11}\frac{\partial u}{\partial x} + a_{12}\frac{\partial u}{\partial y} + b_{11}\frac{\partial v}{\partial x} + b_{12}\frac{\partial v}{\partial y} + a_1 u + b_1 v &= f_1, \\ a_{21}\frac{\partial u}{\partial x} + a_{22}\frac{\partial u}{\partial y} + b_{21}\frac{\partial v}{\partial x} + b_{22}\frac{\partial v}{\partial y} + a_2 u + b_2 v &= f_2. \end{aligned}$$

can be transformed into a Vekua equation for the complex valued function w and complex coefficients a, b

$$\frac{\partial w}{\partial \bar{\zeta}} = a(\zeta)w + b(\zeta)\bar{w}, \tag{0.1}$$

where the necessary coordinate transform $\zeta = \zeta(z)$ has to satisfy the Beltrami equation

$$\frac{\partial \zeta}{\partial \bar{z}} = q(z)\frac{\partial \zeta}{\partial z}. \tag{0.2}$$

For mapping one domain to another one by this coordinate transform we need to state boundary conditions. In [1] the Dirichlet problem for the Beltrami equation was generalized to the case of functions with values in a Clifford algebra by using a generalized Cauchy-Riemann operator and its adjoint instead of the corresponding complex Cauchy-Riemann operators. The existence of a unique solution of the problem and a representation formula for the solution were proved.

Another type of boundary value problems for the Beltrami equation in complex analysis are Schwarz problems given by

$$\begin{aligned} \frac{\partial \zeta}{\partial \bar{z}} &= q(z)\frac{\partial \zeta}{\partial z} \\ \operatorname{Re}\,\zeta(z) &= g(z), \quad z \in \partial\Omega \\ \operatorname{Im}\,\zeta(z_0) &= c, \qquad z_0 \in \bar{\Omega}. \end{aligned}$$

The existence of a solution to the Schwarz problem was proved in [2] by applying a fixed-point iteration. In the talk we will discuss the mentioned results for Dirichlet problem and we will also generalize the Schwarz type problems to the higher-dimensional case. Finally we discuss the relations between the two boundary value problems. The three-dimensional case is of special interest due to its practical relevance and because it is not covered by the mentioned papers. An outlook for considering these problems in the framework of discrete function theories will be given.

[1] K. Gürlebeck and U. Yüksel (2018), *On a Dirichlet Problem for a Generalized Beltrami Equation*, Adv. Appl. Clifford Algebras, in press.

[2] U. Yksel (2011): *A Schwarz problem for the generalized Beltrami equation*, Complex Variables and Elliptic Equations: An International Journal, 56:6, 503-511

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