SEMINARIO DE MATEMÁTICAS

Departamento Académico de Matemáticas del Itam

Set-theoretic aspects of Riemann's rearrangement theorem

Dr. Jörg Brendle

Graduate School of System Informatics, Kobe University

Abstract. Riemann's rearrangement theorem states that given a conditionally convergent series $\sum_n a_n$ and any real number r, there is a rearrangement of the series converging to r, that is, $\sum_n a_{p(n)} = r$ where $p \in \mathrm{Sym}(\mathbb{N})$ is a permutation of the natural numbers $\mathbb{N}$. Also, there are permutations $p \in \mathrm{Sym}(\mathbb{N})$ such that $\sum_n a_{p(n)}$ diverges, either to $+\infty$ or to $-\infty$ or by oscillation. Looking at Riemann's theorem from the point of view of set theory, we may ask:

- How many permutations are needed so that for every conditionally convergent series $\sum_n a_n$, one of these permutations p makes $\sum_n a_{p(n)}$ different from $\sum_n a_n$, in the sense that $\sum_n a_{p(n)}$ either diverges or converges to a value different from $\sum_n a_n$?
- How many permutations are needed so that for every conditionally convergent series $\sum_n a_n$, $\sum_n a_{p(n)}$ converges to a value different from $\sum_n a_n$ for one of these permutations p?

These questions naturally lead to cardinal invariants of the continuum, that is, cardinal numbers describing the structure of the real numbers and taking values between the first uncountable cardinal $\aleph_1$ and the cardinality of the continuum $\mathfrak{c}$. In my talk, I will present basics about cardinal invariants and an outline of the closely related forcing theory. I will then come back to Riemann's theorem, give answers to the above questions, and close with some open problems.

VIERNES 16 DE AGOSTO DE 2019, 13:00 HRS.

SALÓN B5, CAMPUS RÍO HONDO

ITAM