# CENTRALIZER CLONES: BLESSING OR CURSE OF FINITENESS?

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We give an introduction to structures arising from the composition of finitary functions on finite sets, called *clones*. These can be seen as multivariate generalizations of permutation groups and transformation monoids and thus are able to capture similar to automorphism groups and endomorphism monoids "higher-ary" symmetries of mathematical structures. For instance, algebraic structures like groups, rings, lattices, or relational structures like graphs, digraphs or hypergraphs or topological spaces and many others can be considered in this respect, and the "higher-ary" symmetries represented in their clone can encode valuable structural information, even if the endomorphism monoid (the unary part) is trivial, that is, the structure has no ordinary symmetries.

In fact, part of the composition structure of morphisms in any concrete category (over the category of sets) with finite powers gives rise to clones. In particular, starting with a category of algebraic structures, for instance, a variety, like groups or lattices or Boolean algebras, one gets clones of homomorphisms, that are called *centralizer clones*, which are determined by some generalized commutation condition in analogy to group theory. Centralizer clones enjoy some fundamentally different properties as compared to ordinary clones, which might make the theory of all centralizer clones more accessible than the whole of clone theory. On the other hand not too much is known about these special clones.

One particular application of clones lies in the complexity analysis of computational decision problems in theoretical computer science, known as constraint satisfaction problems (CSPs), which, in different variants, have wide ranging applications. Recently, the algebraic theory of clones has led to a breakthrough in the complexity classification of all CSPs on finite sets by verifying a 30-year old conjecture stating that any such problem is either easy (solvable in polynomial time) or hard (NP-complete). In this context it is known that it is sufficient to prove such a dichotomy only for all centralizer clones (to have it for all clones), and while ordinary CSPs are inherently relational, the CSPs corresponding to centralizer clones are notably different in that they correspond to functional problems (systems of equations). Hence, a more thorough understanding of centralizer clones could lead to a new and completely different proof of the CSP dichotomy theorem.

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